

Appendix

A Orientation Gridding

Some of the methods that we describe below represent a discrete multi-modal orientation distribution. To obtain such a distribution, we need to obtain a uniform sampling of orientations. Because we are designing an orientation estimation method for use in an object manipulation context, the objects may appear at any arbitrary orientation. Thus, we want to estimate the object’s orientation distribution over the entire $SO(3)$ orientation space. As such, our orientation gridding must cover this entire space.

In [1], a uniform gridding of Euler angles is used, but as shown in [2], this does not produce a uniform sampling of $SO(3)$. Another approach is to use a uniform random sampling of quaternions; however, we desire a deterministic method to ensure uniform coverage of the orientation space and for repeatability.

To generate a uniform grid of orientations over $SO(3)$, we note that an orientation quaternion is a 4-dimensional unit vector, and thus orientations can be represented as points on a unit 3-sphere. By embedding the “600-cell”, a convex regular 4-polytope whose 600 ”faces” consist of tetrahedra, into the 3-sphere, we can generate an arbitrarily fine gridding through successive tetrahedral subdivision [3]. In our experiments, we subdivide each tetrahedra in the 600-cell twice, leading to a set of tetrahedra with 3885 unique vertices. This gridding gives us a maximum distance from any orientation to its closest vertex of about 10 degrees.

B Normalizing Histogram Distribution

Given an unnormalized distribution with a value of f_i at vertex x_i over the half 3-sphere, whose area is π^2 , the normalized distribution is given by $\frac{1}{\eta} f_i$ where

$$\eta = \frac{\pi^2}{N} \sum_i^N f_i$$

If you look at single nearest neighbor interpolation, this breaks the half sphere into N equi-area regions, each of which has a uniform value of $\frac{1}{\eta} f_i$ and surface area of $\frac{\pi^2}{N}$. The surface integral this interpolation over the half 3-sphere is

$$\begin{aligned} \iint_{\mathcal{S}} p_1(x) d\mathcal{S} &= \sum_i^N \frac{\pi^2}{N} \frac{1}{\eta} f_i \\ &= \frac{1}{\frac{\pi^2}{N} \sum_i^N f_i} \frac{\pi^2}{N} \sum_i^N f_i \\ &= 1 \end{aligned}$$

K nearest neighbor smoothing with inverse distance weighting $d(\mathbf{q}_1, \mathbf{q}_2)$ is then a convex combination of the normalized values,

$$\begin{aligned} p_K(\mathbf{q}^*) &= \frac{\sum_k \frac{1}{\eta} f_k d(\mathbf{q}^*, \mathbf{q}_k)}{\sum_k d(\mathbf{q}^*, \mathbf{q}_k)} \\ &= \frac{N}{\pi^2 \sum_i^N f_i} \frac{\sum_k f_k d(\mathbf{q}^*, \mathbf{q}_k)}{\sum_k d(\mathbf{q}^*, \mathbf{q}_k)} \end{aligned}$$

C K Nearest Neighbor Loss

Using the normalization described above, log likelihood of \mathbf{q}^* is

$$\begin{aligned}\mathcal{L}_K(\mathbf{q}^*) &= \log(p_K(\mathbf{q}^*)) \\ &= \log\left(\frac{N}{\pi^2 \sum_i^N f_i} \frac{\sum_k f_k d(\mathbf{q}^*, \mathbf{q}_k)}{\sum_k d(\mathbf{q}^*, \mathbf{q}_k)}\right) \\ &= \log\left(\frac{N}{\pi^2}\right) - \log\left(\sum_i^N f_i\right) + \log\left(\sum_k f_k d(\mathbf{q}^*, \mathbf{q}_k)\right) - \log\left(\sum_k d(\mathbf{q}^*, \mathbf{q}_k)\right)\end{aligned}$$

Removing the terms that are constant with respect to the weights leaves you with

$$\mathcal{L}_K(\mathbf{q}^*) \propto -\log\left(\sum_i^N f_i\right) + \log\left(\sum_k f_k d(\mathbf{q}^*, \mathbf{q}_k)\right)$$

D Bingham Parameterization

The Quaternion rotation parameter \mathbf{M} of a Bingham distribution can be decomposed using Cayley's factorization into $\mathbf{q}_L = [a, b, c, d]$ and $\mathbf{q}_R = [p, q, r, s]$, such that

$$\mathbf{M} = \begin{bmatrix} a & -b & -c & -d \\ b & a & -d & c \\ c & d & a & -b \\ d & -c & b & a \end{bmatrix} \begin{bmatrix} p & -q & -r & -s \\ q & p & s & -r \\ r & -s & p & q \\ s & r & -q & p \end{bmatrix}$$

To simplify this representation to distribution centered on $\bar{\mathbf{q}}$, the column associated with the largest value in \mathbf{Z} must be correctly aligned to the column of \mathbf{M} containing the mean quaternion $\bar{\mathbf{q}}$. One possible solution to this is to have $\mathbf{q}_L = \bar{\mathbf{q}}$ and $\mathbf{q}_R = [1, 0, 0, 0]$. This results in the desired effect, with the second matrix in equation [above] becoming the identity.

To rotated that distribution about the mean quaternion, $\bar{\mathbf{q}}$, you can pose multiply but the three dimensional rotation \mathbf{R} defined by quaternion $\mathbf{q}_{rot} = [x, y, z, w]$

$$\mathbf{R} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & x^2 + y^2 - z^2 - w^2 & 2 * (y * z - x * w) & 2 * (y * w + x * z) \\ 0 & 2 * (y * z + x * w) & x^2 - y^2 + z^2 - w^2 & 2 * (z * w - x * y) \\ 0 & 2 * (y * w - x * z) & 2 * (z * w + x * y) & x^2 - y^2 - z^2 + w^2 \end{bmatrix}$$

E Symmetric Angular Error

For symmetric objects, standard rotational error is not a meaningful metric. However, given the symmetries of the object, a minimum angular distance between the estimated orientation $\hat{\mathbf{q}}$ and the true orientation \mathbf{q} over all possible symmetric rotations can be calculated.

For the case of discrete rotational symmetries, like in a block, we can compute a symmetry-aware angular distance by iterating over each rotation \mathbf{p} in the set of symmetric rotations Φ , applying it to the quaternion \mathbf{q} and computing the distance to $\hat{\mathbf{q}}$, as $\theta = \min_{\mathbf{p} \in \Phi} \cos^{-1}(\|\mathbf{q}\mathbf{p} \cdot \hat{\mathbf{q}}\|)$.

For continuous rotational symmetries, as in a bowl, we perform the following computation: given the axis of symmetry ξ , we let \mathbf{x} be a vector describing this axis of symmetry ξ rotated by the ground truth quaternion q , computed as $\mathbf{x} = \mathbf{q}\xi\mathbf{q}^{-1}$. Similarly, let $\hat{\mathbf{x}}$ be a vector describing ξ rotated by $\hat{\mathbf{q}}$, computed as $\hat{\mathbf{x}} = \hat{\mathbf{q}}\xi\hat{\mathbf{q}}^{-1}$. Then we can compute a symmetry-aware angular distance between \mathbf{q} and $\hat{\mathbf{q}}$ as $\theta = \cos^{-1}(\mathbf{x} \cdot \hat{\mathbf{x}})$.

Objects	Histogram					Bingham				
	Uniform	Conf	Reg	Comp	Cosine	Fixed	Mix	Dropout	Reg Iso	Reg Full
Non-Symmetric-DF	23.07	85.17	73.07	72.03	65.93	87.94	87.94	81.14	87.93	87.93
Symmetric-DF	51.62	63.37	55.70	52.76	53.93	60.34	60.34	50.90	60.35	60.35
Non-Symmetric-PC	23.06	68.16	79.93	80.62	84.37	85.88	-	86.38	85.88	85.88
Symmetric-PC	52.18	66.18	77.33	76.14	80.31	77.79	-	76.84	77.79	77.79
All	27.73	67.84	79.51	79.90	83.71	84.57	-	84.84	84.57	84.57

Table 1: AUC of Symmetric Angular Error of Distribution Mode

F Individual Object Results

Objects	Histogram					Bingham				
	Uniform	Conf	Reg	Comp	Cosine	Fixed	Mix	Dropout	Reg Iso	Reg Full
master chef can	-2.29	-0.78	-2.82	1.01	-1.91	-0.97	-0.94	-1.04	-1.66	1.21
cracker box	-2.29	4.03	-1.00	2.26	-1.91	1.13	1.68	1.14	3.75	1.81
sugar box	-2.29	3.77	2.64	2.24	-1.80	0.04	0.97	1.95	5.94	-0.06
tomato soup can	-2.29	0.90	-2.24	1.61	-1.73	-0.27	-0.02	1.16	2.02	1.63
mustard bottle	-2.29	3.85	-1.21	2.15	-1.73	2.38	2.41	1.12	4.61	1.73
tuna fish can	-2.29	-3.12	-5.86	0.42	-2.05	-1.17	-2.19	-0.96	-0.20	0.31
pudding box	-2.29	2.18	-0.18	-1.35	-1.95	1.15	1.19	0.82	2.64	0.47
gelatin box	-2.29	4.65	2.48	2.08	-1.58	1.33	1.92	1.25	6.25	2.37
potted meat can	-2.29	1.18	-1.36	0.61	-1.91	-0.01	0.03	0.95	3.28	-0.91
banana	-2.29	2.58	-1.12	1.03	-1.90	-0.65	-0.45	0.11	0.58	-0.86
pitcher base	-2.29	3.34	2.13	2.28	-1.79	2.82	3.11	1.76	4.68	1.29
bleach cleanser	-2.29	3.91	-4.19	1.76	-1.85	1.56	2.42	1.24	4.70	1.77
bowl	-2.29	1.37	-8.88	-0.46	-2.13	-2.45	-2.33	-8.99	-2.77	-0.28
mug	-2.29	3.73	-6.18	-1.03	-2.21	0.37	0.65	1.69	2.50	1.94
power drill	-2.29	4.31	0.40	1.91	-1.94	3.14	3.25	1.22	5.92	1.57
wood block	-2.29	2.64	-6.22	-2.14	-2.21	-2.13	-2.15	-13.00	-2.09	0.35
scissors	-2.29	3.74	-13.32	-0.29	-2.15	0.73	0.82	-2.15	0.51	1.44
large marker	-2.29	-7.59	-11.48	0.21	-2.03	-1.87	-1.55	-0.73	-0.29	0.76
large clamp	-2.29	-5.64	-13.67	-10.45	-2.32	-2.45	-2.33	-5.66	-6.67	-2.85
extra large clamp	-2.29	-5.20	-11.94	-7.49	-2.19	-2.24	-2.27	-9.28	-2.92	-2.02
foam brick	-2.29	-0.26	-12.89	-2.34	-2.29	-2.31	-2.18	-6.06	-2.28	-1.98
All	-2.29	0.86	-3.90	0.09	-1.95	-0.01	0.18	-0.74	1.71	0.57

Table 2: Mean Log Likelihood of Ground Truth Orientation using DenseFusion

Objects	Histogram					Bingham				
	Uniform	Conf	Reg	Comp	Cosine	Fixed	Mix	Dropout	Reg Iso	Reg Full
master chef can	-2.29	-4.99	1.40	2.32	-0.83	-2.29	-	0.09	-1.57	-1.70
cracker box	-2.29	-6.06	0.92	-0.09	-1.06	-2.29	-	3.71	0.90	1.81
sugar box	-2.29	0.77	3.19	2.62	-0.82	-2.28	-	4.20	4.63	3.17
tomato soup can	-2.29	-0.21	2.14	2.52	-0.82	-2.29	-	3.99	1.78	3.45
mustard bottle	-2.29	-1.18	1.28	3.02	-0.79	-2.29	-	4.81	4.39	3.39
tuna fish can	-2.29	-4.24	0.91	2.64	-1.12	-2.29	-	1.23	-0.12	0.74
pudding box	-2.29	-1.62	2.97	3.13	-0.95	-2.29	-	4.54	4.12	2.65
gelatin box	-2.29	-0.56	3.11	3.53	-0.93	-2.28	-	5.73	5.88	3.92
potted meat can	-2.29	-2.32	1.59	1.60	-0.96	-2.29	-	3.06	0.75	2.01
banana	-2.29	-2.01	2.52	2.27	-1.12	-2.28	-	3.70	3.06	2.93
pitcher base	-2.29	-0.64	2.74	2.35	-0.75	-2.29	-	4.88	3.86	2.96
bleach cleanser	-2.29	-4.31	1.80	2.29	-0.88	-2.28	-	3.38	2.86	1.78
bowl	-2.29	-9.73	-1.95	-1.21	-1.96	-2.29	-	-9.62	-13.05	-5.21
mug	-2.29	-2.17	2.46	2.75	-0.69	-2.28	-	4.72	2.51	2.64
power drill	-2.29	-1.44	2.53	2.43	-0.84	-2.29	-	4.17	4.24	3.41
wood block	-2.29	-5.01	1.09	-0.51	-1.44	-2.28	-	4.49	0.93	0.03
scissors	-2.29	-4.56	-1.15	0.66	-1.25	-2.29	-	1.63	-1.25	0.85
large marker	-2.29	-3.37	0.65	1.02	-1.31	-2.29	-	-8.13	-1.69	0.39
large clamp	-2.29	-9.34	-3.36	-1.63	-1.59	-2.29	-	-3.54	-7.28	-5.02
extra large clamp	-2.29	-6.59	0.15	0.19	-1.31	-2.29	-	-5.03	-10.13	-2.56
foam brick	-2.29	-5.75	0.14	1.70	-1.42	-2.29	-	-12.06	-11.84	-4.10
All	-2.29	-3.31	1.37	1.74	-1.02	-2.29	-	1.43	0.20	1.11

Table 3: Mean Log Likelihood of Ground Truth Orientation using PoseCNN

Objects	Histogram					Bingham				
	Uniform	Conf	Reg	Comp	Cosine	Fixed	Mix	Dropout	Reg Iso	Reg Full
master chef can	0.00	65.45	56.85	56.36	49.81	68.89	68.89	12.75	68.87	68.87
cracker box	0.00	80.56	52.27	64.08	61.75	83.78	83.78	4.40	83.90	83.90
sugar box	5.85	87.75	80.27	80.42	76.32	91.01	91.01	10.24	91.04	91.04
tomato soup can	27.78	82.05	79.41	82.80	80.37	84.48	84.48	15.47	84.49	84.49
mustard bottle	0.81	88.23	80.18	80.47	79.42	91.92	91.92	13.79	91.76	91.76
tuna fish can	50.86	77.78	72.51	74.24	70.42	79.94	79.94	19.50	79.93	79.93
pudding box	18.17	81.42	65.09	58.21	71.22	85.86	85.86	23.67	86.12	86.12
gelatin box	66.26	94.15	89.79	88.99	90.87	95.96	95.96	28.95	95.95	95.95
potted meat can	33.33	81.30	73.26	72.50	70.29	82.43	82.43	12.74	82.48	82.48
banana	35.84	72.36	33.79	50.80	33.90	74.67	74.67	6.85	74.69	74.69
pitcher base	0.00	84.51	72.57	71.63	46.64	89.80	89.80	4.97	89.73	89.73
bleach cleanser	0.00	84.41	70.55	69.58	53.11	87.88	87.88	6.24	87.74	87.74
bowl	31.14	26.91	31.98	18.22	11.45	4.65	4.65	1.80	4.65	4.65
mug	26.32	88.24	78.18	42.42	58.63	90.33	90.33	9.03	90.44	90.44
power drill	10.36	87.38	79.35	69.23	55.29	90.75	90.75	6.23	90.76	90.76
wood block	0.00	74.19	7.68	9.52	12.60	32.79	32.79	0.00	32.79	32.79
scissors	40.05	80.58	64.39	44.00	29.82	82.03	82.03	10.05	82.10	82.10
large marker	59.87	85.64	78.42	72.10	62.25	89.33	89.33	30.28	89.34	89.34
large clamp	15.79	19.55	14.81	12.86	9.71	13.03	13.03	1.43	13.02	13.02
extra large clamp	15.24	21.67	9.64	17.18	15.21	23.50	23.50	0.19	23.59	23.59
foam brick	48.04	55.88	49.30	54.08	49.02	51.68	51.68	19.75	51.73	51.73
All	20.19	73.72	62.92	61.33	55.72	74.50	74.50	10.77	74.51	74.51

Table 4: AUC of Average Distance Error of Distribution Mode using DenseFusion

Objects	Histogram					Bingham				
	Uniform	Conf	Reg	Comp	Cosine	Fixed	Mix	Dropout	Reg Iso	Reg Full
master chef can	0.00	12.02	52.17	52.62	60.45	50.20	-	49.90	50.20	50.20
cracker box	0.00	29.15	41.51	38.05	42.96	53.01	-	55.11	53.01	53.01
sugar box	4.04	60.35	65.23	60.20	65.96	68.47	-	68.12	68.47	68.47
tomato soup can	22.13	67.84	66.60	66.79	68.82	68.19	-	69.10	68.19	68.19
mustard bottle	0.47	53.49	76.64	78.30	79.34	81.09	-	80.00	81.09	81.09
tuna fish can	44.45	68.03	66.87	70.62	70.34	70.70	-	71.24	70.70	70.70
pudding box	10.87	38.41	60.08	62.21	60.96	62.56	-	62.65	62.56	62.56
gelatin box	59.30	68.56	73.61	74.21	73.61	75.19	-	75.18	75.19	75.19
potted meat can	25.91	57.76	57.24	61.33	57.87	60.52	-	60.66	60.52	60.52
banana	30.27	52.46	65.11	67.60	72.62	72.39	-	73.42	72.39	72.39
pitcher base	0.00	23.00	50.60	46.98	52.26	53.29	-	54.01	53.29	53.29
bleach cleanser	0.00	37.20	44.49	45.91	48.99	50.43	-	50.68	50.43	50.43
bowl	16.72	10.37	5.05	4.82	7.88	3.26	-	5.53	3.26	3.26
mug	16.18	48.29	52.01	57.58	57.80	58.49	-	59.41	58.49	58.49
power drill	9.19	40.20	53.39	51.46	52.75	55.23	-	55.08	55.23	55.23
wood block	0.00	12.23	15.79	4.56	8.02	27.08	-	29.76	27.08	27.08
scissors	17.03	30.88	28.39	28.17	32.75	35.74	-	35.97	35.74	35.74
large marker	43.03	54.66	57.15	58.25	58.40	58.09	-	58.38	58.09	58.09
large clamp	17.03	26.86	29.12	29.70	29.20	25.24	-	25.86	25.24	25.24
extra large clamp	11.39	12.86	20.92	16.95	20.48	18.52	-	18.26	18.52	18.52
foam brick	42.60	50.90	40.79	55.96	55.55	40.25	-	40.75	40.25	40.25
All	15.87	43.19	51.49	51.67	54.06	54.34	-	54.81	54.34	54.34

Table 5: AUC of Average Distance Error of Distribution Mode using PoseCNN

Objects	Histogram					Bingham				
	Uniform	Conf	Reg	Comp	Cosine	Fixed	Mix	Dropout	Reg Iso	Reg Full
master chef can	85.00	93.89	92.88	93.27	93.09	94.39	94.39	40.20	94.41	94.41
cracker box	70.90	90.66	85.45	86.78	86.97	91.73	91.73	22.71	91.78	91.78
sugar box	77.29	94.08	91.50	91.35	89.80	95.34	95.34	31.61	95.39	95.39
tomato soup can	88.35	95.15	93.92	94.95	94.77	95.70	95.70	33.12	95.69	95.69
mustard bottle	79.79	95.12	92.27	92.38	92.20	96.30	96.30	41.88	96.26	96.26
tuna fish can	90.83	95.60	94.54	95.34	93.73	96.00	96.00	44.76	95.99	95.99
pudding box	87.00	92.54	88.75	87.66	89.27	93.90	93.90	47.31	93.92	93.92
gelatin box	88.84	96.69	94.66	94.70	95.34	97.35	97.35	52.04	97.35	97.35
potted meat can	85.13	91.27	90.16	90.01	89.75	91.60	91.60	27.29	91.60	91.60
banana	70.34	92.73	91.08	91.62	87.47	93.18	93.18	29.76	93.18	93.18
pitcher base	70.68	92.90	90.73	89.77	83.33	94.83	94.83	22.23	94.82	94.82
bleach cleanser	68.16	93.04	89.90	89.35	82.71	94.72	94.72	25.25	94.69	94.69
bowl	73.64	85.96	85.87	84.30	81.34	86.83	86.83	32.91	86.85	86.85
mug	90.20	96.09	94.66	90.46	91.28	96.54	96.54	21.70	96.57	96.57
power drill	68.30	93.75	90.48	85.52	81.65	94.94	94.94	21.94	94.94	94.94
wood block	74.66	89.96	78.24	76.93	75.96	90.42	90.42	16.36	90.40	90.40
scissors	72.39	92.66	89.81	83.82	82.56	92.58	92.58	38.16	92.59	92.59
large marker	77.02	93.64	90.74	87.88	86.59	95.58	95.58	49.34	95.65	95.65
large clamp	42.42	47.48	47.41	45.93	45.21	47.72	47.72	18.24	47.85	47.85
extra large clamp	61.12	71.20	69.69	69.03	66.74	71.36	71.36	15.62	71.43	71.43
foam brick	90.62	91.62	91.76	92.00	91.35	92.79	92.79	57.61	92.82	92.82
All	76.75	90.02	87.84	87.07	85.35	90.89	90.89	31.08	90.91	90.91

Table 6: AUC of Average Symmetric Distance Error of Distribution Mode using DenseFusion

Objects	Histogram					Bingham				
	Uniform	Conf	Reg	Comp	Cosine	Fixed	Mix	Dropout	Reg Iso	Reg Full
master chef can	76.53	76.65	84.03	83.83	84.12	83.80	-	84.03	83.80	83.80
cracker box	63.76	68.56	72.49	72.61	73.00	76.65	-	77.35	76.65	76.65
sugar box	71.42	82.09	82.98	82.67	83.12	84.03	-	83.86	84.03	84.03
tomato soup can	78.92	83.31	83.26	83.25	83.42	83.43	-	83.58	83.43	83.43
mustard bottle	78.03	87.63	89.68	89.97	90.28	90.72	-	90.66	90.72	90.72
tuna fish can	85.99	87.86	88.42	88.40	88.21	88.12	-	88.36	88.12	88.12
pudding box	75.24	76.58	78.26	78.67	78.35	78.94	-	78.88	78.94	78.94
gelatin box	82.21	84.64	85.35	85.63	85.41	85.96	-	85.91	85.96	85.96
potted meat can	79.02	80.82	80.62	80.95	80.66	81.04	-	81.06	81.04	81.04
banana	67.58	81.61	83.55	85.22	86.39	86.39	-	86.94	86.39	86.39
pitcher base	64.38	71.25	77.46	77.94	77.86	78.17	-	78.65	78.17	78.17
bleach cleanser	56.84	69.86	72.22	71.82	72.16	73.05	-	73.00	73.05	73.05
bowl	71.50	69.61	72.47	73.30	72.64	70.51	-	70.19	70.51	70.51
mug	76.65	77.80	78.33	78.44	78.51	78.32	-	78.57	78.32	78.32
power drill	58.22	68.59	72.47	71.15	71.78	72.95	-	72.78	72.95	72.95
wood block	53.44	63.35	65.14	63.32	64.74	62.86	-	63.94	62.86	62.86
scissors	49.20	55.95	55.61	54.70	56.99	57.92	-	57.48	57.92	57.92
large marker	62.19	70.13	69.95	71.13	71.26	70.96	-	71.21	70.96	70.96
large clamp	46.49	50.72	52.67	52.51	51.63	52.28	-	51.81	52.28	52.28
extra large clamp	47.93	50.27	53.69	53.62	54.40	53.52	-	54.58	53.52	53.52
foam brick	85.72	86.00	86.88	86.88	86.92	86.74	-	86.77	86.74	86.74
All	68.73	74.32	76.47	76.43	76.64	77.02	-	77.16	77.02	77.02

Table 7: AUC of Average Symmetric Distance Error of Distribution Mode using PoseCNN

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